IJDCST @Sep, Issue- V-2, I-6, SW-10
ISSN-2320-7884 (Online)
ISSN-2321-0257 (Print)

Poisson to Normal Approximation through Diagrammatic approach

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ABSTRACT

ICT based learning, the inter relationship of various probability distributions can be studied viz. shape, location and scale parameters through visual diagrammatic methods. In this paper we have discussed the approximation of Poisson to Normal probability distribution.

1. INTRODUCTION

Information and Communication Technologies (ICT) is becoming increasingly the necessary integrated information platform in teaching and learning processes. Implementing ICT can bring benefits to learners, at the same time it provides a broad view of the some of the complex concepts. ICTs are making dynamic changes in class room environment. ICTs provide both students and teachers with more productive learning opportunities to needs of the society. ICTs greatly facilitate the acquisition and absorption of knowledge in the interested field.

Task: Students of statistics should learn discrete and continuous probability distributions in probability theory. Theoretically the students should learn to go to great lengths of derivations relating to the distributions and their interrelationships. Each distribution has its own unique behavioral pattern and characteristics. Of the all existing distributions Normal Distribution plays a very important role in statistical theory.

2. NORMAL DISTRIBUTION

The normal distribution is the most important and also widely used distribution in statistics. The Normal distribution is also the core of the space of all observable processes. This distribution often provides a reasonable approximation to variety of data. It is sometimes called the "bell curve," although the tonal qualities of such a bell would be less than pleasing. It is also called the "Gaussian curve" after the mathematician Karl Friedrich Gauss.

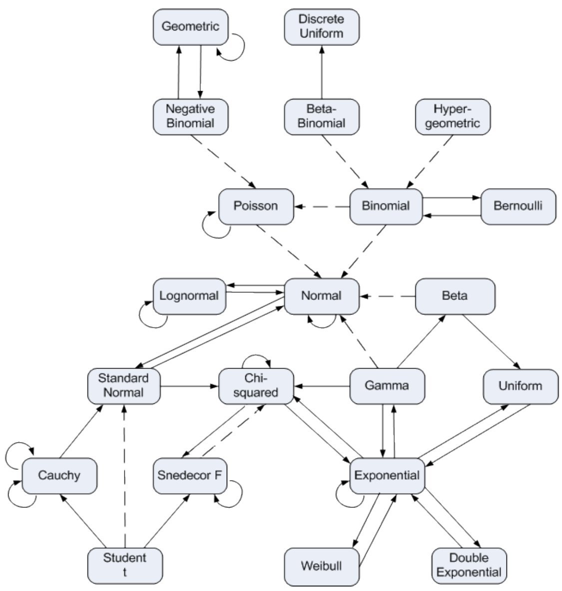
The density of the normal distribution is

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

The parameters μ and σ are the mean and standard deviation, respectively, and define the normal distribution. The symbol *e* is the base of the natural logarithm and π is the constant pi.

3. DIAGRAM OF DISTRIBUTION RELATIONSHIPS

Probability distributions have surprising number inter-connections. A dashed line in the chart below indicates an approximate (limit) relationship between two distribution families. A solid line indicates an exact relationship: special case, sum, or transformation.



ISSN-2320-7884 (Online)
ISSN-2321-0257 (Print)

Fig. 1 Inter relationship of various probability distributions with Normal

4. PARAMETERIZATIONS

The precise relationships between distributions depend on parameterization. The relationships detailed below depend on the following parameterizations for the PDFs.

Let C(n, k) denote the binomial coefficient(n, k) and
$B(a, b) = \Gamma(a)\,\Gamma(b) / \Gamma(a + b)$.

Geometric: $f(x) = p\,(1-p)^x$ for non-negative integers x.

Discrete uniform: $f(x) = 1/n$ for $x = 1, 2, ..., n$.

Negative binomial: $f(x) = C(r + x - 1, x)\, p^r(1-p)^x$ for non-negative integers x.

Beta binomial: $f(x) = C(n, x)\, B(\alpha + x, n + \beta - x) / B(\alpha, \beta)$ for $x = 0, 1, ..., n$.

Hypergeometric: $f(x) = C(M, x)\, C(N-M, K - x) / C(N, K)$ for $x = 0, 1, ..., N$.

Poisson: $f(x) = \exp(-\lambda)\, \lambda^x / x!$ for non-negative integers x. The parameter λ is both the mean and the variance.

Binomial: $f(x) = C(n, x)\, p^x(1 - p)^{n-x}$ for $x = 0, 1, ..., n$.

Bernoulli: $f(x) = p^x(1 - p)^{1-x}$ where x = 0 or 1.

Lognormal: $f(x) = (2\pi\sigma^2)^{-1/2} \exp(-(\log(x) - \mu)^2 / 2\sigma^2) / x$ for positive x. Note that μ and σ^2 are not the mean and variance of the distribution.

Normal: $f(x) = (2\pi\, \sigma^2)^{-1/2} \exp(- ½((x - \mu)/\sigma)^2)$ for all x.

Beta: $f(x) = \Gamma(\alpha + \beta)\, x^{\alpha-1}(1 - x)^{\beta-1} / (\Gamma(\alpha)\, \Gamma(\beta))$ for $0 \le x \le 1$.

Standard normal: $f(x) = (2\pi)^{-1/2} \exp(-x^2/2)$ for all x.

Chi-squared: $f(x) = x^{-\nu/2-1} \exp(-x/2) / \Gamma(\nu/2)\, 2^{\nu/2}$ for positive x. The parameter ν is called the degrees of freedom.

Gamma: $f(x) = \beta^{-\alpha} x^{\alpha-1} \exp(-x/\beta) / \Gamma(\alpha)$ for positive x. The parameter α is called the shape and β is the scale.

Uniform: $f(x) = 1$ for $0 \le x \le 1$.

Cauchy: $f(x) = \sigma/(\pi((x - \mu)^2 + \sigma^2))$ for all x. Note that μ and σ are location and scale parameters. The Cauchy distribution has no mean or variance.

Snedecor F: $f(x)$ is proportional to $x^{(\nu_1 - 2)/2} / (1 + (\nu_1/\nu_2)\, x)^{(\nu_1 + \nu_2)/2}$ for positive x.

Exponential: $f(x) = \exp(-x/\mu)/\mu$ for positive x. The parameter μ is the mean.

Student t: $f(x)$ is proportional to $(1 + (x^2/\nu))^{-(\nu + 1)/2}$ for positive x. The parameter ν is called the degrees of freedom.

Weibull: $f(x) = (\gamma/\beta)\, x^{\gamma-1} \exp(- x^{\gamma}/\beta)$ for positive x. The parameter γ is the shape and β is the scale.

Double exponential: $f(x) = \exp(-|x-\mu|/\sigma) / 2\sigma$ for all x. The parameter μ is the location and mean; σ is the scale.

5. RELATIONSHIPS

In all statements about two random variables, the random variables are implicitly independent.

Geometric / negative binomial: If each X_i is geometric random variable with probability of success p than the sum of n X_i's is a negative binomial random variable with parameters n and p.

Negative binomial / geometric: A negative binomial distribution with r = 1 is a geometric distribution.

Negative binomial / Poisson: If X has a negative binomial random variable with r large, p near 1, and $r(1-p) = \lambda$, then $F_X \approx F_Y$ where Y is a Poisson random variable with mean λ.

ISSN-2320-7884 (Online)
ISSN-2321-0257 (Print)

Beta-binomial / discrete uniform: A beta-binomial (n, 1, 1) random variable is a discrete uniform random variable over the values 0 ... n.

Beta-binomial / binomial: Let X be a beta-binomial random variable with parameters (n, α, β). Let p = α/(α + β) and suppose α + β is large. If Y is a binomial(n, p) random variable then $F_X \approx F_Y$.

Hypergeometric / binomial: The difference between a hypergeometric distribution and a binomial distribution is the difference between sampling without replacement and sampling with replacement. As the population size increases relative to the sample size, the difference becomes negligible.

Geometric / geometric: If X_1 and X_2 are geometric random variables with probability of success p_1 and p_2 respectively, then $\min(X_1, X_2)$ is a geometric random variable with probability of success $p = p_1 + p_2 - p_1 p_2$. The relationship is simpler in terms of failure probabilities: $q = q_1 q_2$.

Poisson / Poisson: If X_1 and X_2 are Poisson random variables with means μ_1 and μ_2 respectively, then $X_1 + X_2$ is a Poisson random variable with mean $\mu_1 + \mu_2$.

Binomial / Poisson: If X is a binomial(n, p) random variable and Y is a Poisson(np) distribution then $P(X = n) \approx P(Y = n)$ if n is large and np is small.

Binomial / Bernoulli: If X is a binomial(n, p) random variable with n = 1, X is a Bernoulli(p) random variable.

Bernoulli / Binomial: The sum of n Bernoulli(p) random variables is a binomial(n, p) random variable.

Poisson / normal: If X is a Poisson random variable with large mean and Y is a normal distribution with the same mean and variance as X, then for integers j and k, $P(j \le X \le k) \approx P(j - 1/2 \le Y \le k + 1/2)$.

Binomial / normal: If X is a binomial(n, p) random variable and Y is a normal random variable with the same mean and variance as X, i.e. np and np(1-p), then for integers j and k, $P(j \le X \le k) \approx P(j - 1/2 \le Y \le k + 1/2)$. The approximation is better when p ≈ 0.5 and when n is large.

Lognormal / lognormal: If X_1 and X_2 are lognormal random variables with parameters (μ_1, σ_1^2) and (μ_2, σ_2^2) respectively, then $X_1 X_2$ is a lognormal random variable with parameters $(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$.

Normal / lognormal: If X is a normal (μ, σ^2) random variable then e^X is a lognormal (μ, σ^2) random variable. Conversely, if X is a lognormal (μ, σ^2) random variable then log X is a normal (μ, σ^2) random variable.

Beta / normal: If X is a beta random variable with parameters α and β equal and large, $F_X \approx F_Y$ where Y is a normal random variable with the same mean and variance as X, i.e. mean α/(α + β) and variance $\alpha\beta/((\alpha+\beta)^2(\alpha + \beta + 1))$.

Normal / standard normal: If X is a normal(μ, σ^2) random variable then $(X - \mu)/\sigma$ is a standard normal random variable. Conversely, If X is a normal(0,1) random variable then $\sigma X + \mu$ is a normal (μ, σ^2) random variable.

Normal / normal: If X_1 is a normal (μ_1, σ_1^2) random variable and X_2 is a normal (μ_2, σ_2^2) random variable, then $X_1 + X_2$ is a normal $(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ random variable.

Gamma / normal: If X is a gamma(α, β) random variable and Y is a normal random variable with the same mean and variance as X, then $F_X \approx F_Y$ if the shape parameter α is large relative to the scale parameter β.

Gamma / beta: If X_1 is gamma$(\alpha_1, 1)$ random variable and X_2 is a gamma $(\alpha_2, 1)$ random variable then $X_1/(X_1 + X_2)$ is a beta(α_1, α_2) random variable. More generally, if X_1 is gamma(α_1, β_1) random variable and X_2 is gamma(α_2, β_2) random variable then $\beta_2 X_1/(\beta_2 X_1 + \beta_1 X_2)$ is a beta(α_1, α_2) random variable.

Beta / uniform: A beta random variable with parameters α = β = 1 is a uniform random variable.

Chi-squared / chi-squared: If X_1 and X_2 are chi-squared random variables with ν_1 and ν_2 degrees of freedom respectively, then $X_1 + X_2$ is a chi-squared random variable with $\nu_1 + \nu_2$ degrees of freedom.

Standard normal / chi-squared: The square of a standard normal random variable has a chi-squared distribution with one degree of freedom. The sum of the squares of n standard normal random variables is has a chi-squared distribution with n degrees of freedom.

Gamma / chi-squared: If X is a gamma (α, β) random variable with $\alpha = \nu/2$ and $\beta = 2$, then X is a chi-squared random variable with ν degrees of freedom.

Cauchy / standard normal: If X and Y are standard normal random variables, X/Y is a Cauchy(0,1) random variable.

Student t / standard normal: If X is a t random variable with a large number of degrees of freedom ν then $F_X \approx F_Y$ where Y is a standard normal random variable.

Snedecor F / chi-squared: If X is an $F(\nu, \omega)$ random variable with ω large, then ν X is approximately distributed as a chi-squared random variable with ν degrees of freedom.

Chi-squared / Snedecor F: If X_1 and X_2 are chi-squared random variables with ν_1 and ν_2 degrees of freedom respectively, then $(X_1/\nu_1)/(X_2/\nu_2)$ is an $F(\nu_1, \nu_2)$ random variable.

Chi-squared / exponential: A chi-squared distribution with 2 degrees of freedom is an exponential distribution with mean 2.

Exponential / chi-squared: An exponential random variable with mean 2 is a chi-squared random variable with two degrees of freedom.

Gamma / exponential: The sum of n exponential(β) random variables is a gamma(n, β) random variable.

Exponential / gamma: A gamma distribution with shape parameter $\alpha = 1$ and scale parameter β is an exponential(β) distribution.

Exponential / uniform: If X is an exponential random variable with mean λ, then $\exp(-X/\lambda)$ is a uniform random variable. More generally, sticking any random variable into its CDF yields a uniform random variable.

Uniform / exponential: If X is a uniform random variable, $-\lambda \log X$ is an exponential random variable with mean λ. More generally, applying the inverse CDF of any random variable X to a uniform random variable creates a variable with the same distribution as X.

Cauchy reciprocal: If X is a Cauchy (μ, σ) random variable, then 1/X is a Cauchy $(\mu/c, \sigma/c)$ random variable where $c = \mu^2 + \sigma^2$.

Cauchy sum: If X_1 is a Cauchy (μ_1, σ_1) random variable and X_2 is a Cauchy (μ_2, σ_2), then $X_1 + X_2$ is a Cauchy $(\mu_1 + \mu_2, \sigma_1 + \sigma_2)$ random variable.

Student t / Cauchy: A random variable with a t distribution with one degree of freedom is a Cauchy(0,1) random variable.

Student t / Snedecor F: If X is a t random variable with ν degree of freedom, then X^2 is an $F(1,\nu)$ random variable.

Snedecor F / Snedecor F: If X is an $F(\nu_1, \nu_2)$ random variable then 1/X is an $F(\nu_2, \nu_1)$ random variable.

Exponential / Exponential: If X_1 and X_2 are exponential random variables with mean μ_1 and μ_2 respectively, then $\min(X_1, X_2)$ is an exponential random variable with mean $\mu_1 \mu_2/(\mu_1 + \mu_2)$.

Exponential / Weibull: If X is an exponential random variable with mean β, then $X^{1/\gamma}$ is a Weibull(γ, β) random variable.

Weibull / Exponential: If X is a Weibull(1, β) random variable, X is an exponential random variable with mean β.

Exponential / Double exponential: If X and Y are exponential random variables with mean μ, then X-Y is a double exponential random variable with mean 0 and scale μ

Double exponential / exponential: If X is a double exponential random variable with mean 0 and scale λ, then |X| is an exponential random variable with mean λ.

6. USE OF ICT IN ESTABLISHIG RELATIONSHIP

Most of the distributions occurring in practice can be approximated by Normal distribution. More over sampling distributions tend to normality for large samples.

For example, If X ~ Poisson(λ) with λ *large* then X is well approximated by a normal distribution and how large does λ have to be? This can be shown diagrammatically through MATLAB/MATHEMATICA

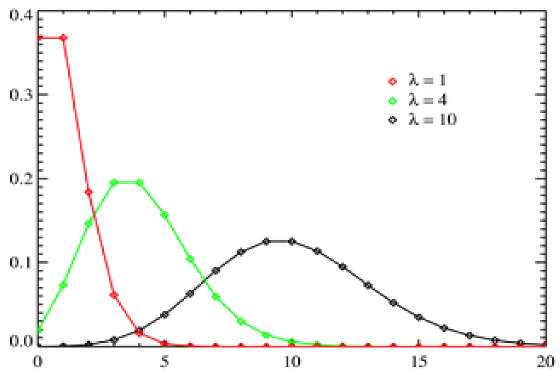


Fig. 1 Poisson approaches to Normal

7. CONCLUSIONS

By drawing the random samples from Poisson distribution and changing the values of the parameter λ such as 1, 4 and10 the shape and location of the distribution can be changed. Therefore ICT enabled teaching gives us an opportunity to study how the approximation to the Normal distribution changes when we alter the parameter of the Poisson distribution. In a similar approach one can demonstrate the inter relationship of various distributions.

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